TNM079: Modeling & Animation

Laboratory report

Splines and Subdivision

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Contents

Abstract

Most common way to represent a line in computer graphics is using curve. The focus of this paper is on represent a line as a curve, curve subdivision and mesh subdivision.

1. Introduction

A curve is a parametric way to show a line. A curve simply can be defined as equation 1.

$$p(t) = \sum_{i=0}^{n} c_i b^i$$

Equation 1 – Represent a curve

In equation 1 c_i represents a set of coefficients and b^i is a set of basic functions.

As mentioned before each line can be represent by a set of curves. So a line in space is divided into different part and each part will be shown by a curve.

- The linear Bezier curve

Linear Bezier curve is the simplest form of a curve (Fig. 1).

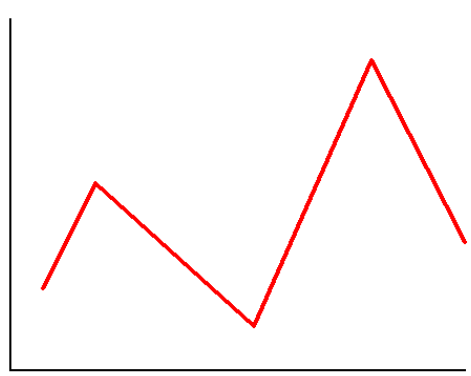

Fig 1: A line which represent with a linear Bezier curve

It is a straight line between two points c_0 and c_1 with a linear interpolation between these two points. It is clear that linear Bezier curve is sharp and harsh.

- The Cubic Bezier curve

Instead of linear Bezier curve, there is another way of represent a curve which is smoother and softer and called cubic Bezier curve (Fig. 2).

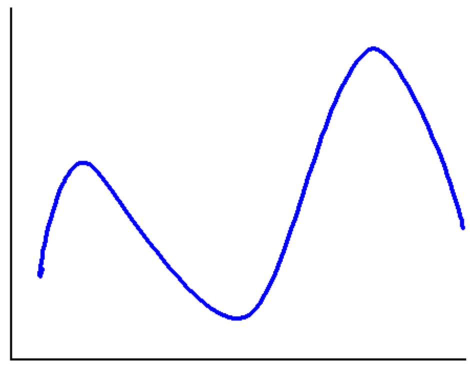

Fig 2: A line which represent with a cubic Bezier curve

2. Method

In this part method of doing two different subdivisions for a curve and mesh will be described step by step:

2-1. Implement of curve subdivision

Curve subdivision is easy and simple. Just enough to add some control point between existing points. To do this following steps should be done:

- The first control point in new set is the same as original one (Equation 2).

$$c'_0 = c_0$$

Equation 2 – The first control point remain without change.

- In a for loop based on size of the all coefficients minus 2 (is it clear why?) the new control points will be calculated by using equation 3 and 4 in order. By doing this, between each two control points; one new control point will be added.

$$c'_i = \frac{1}{8}\,(c_{i-1} + 6c_i + \ c_{i+1})$$

Equation 3 – Calculation of new control points.

$$c'_{i+\frac{1}{2}} = \frac{1}{8}\,(4c_i + c_{i+1})$$

Equation 4 – Calculation of new control points.

The index of $i+\frac{1}{2}$ is referring to this new control point.

- Finally the last point pushes into the set by using equation 5.

$$c'_{end} = \ c_{end}$$

Equation 5 – Calculation of last control point.

The result of curve subdivision is shown in Fig. 3.

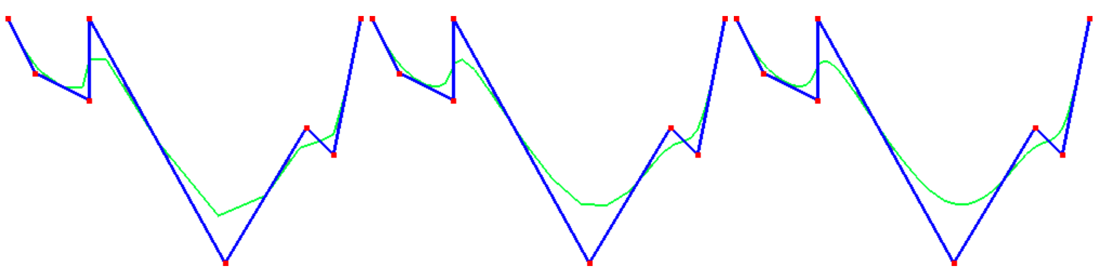

Fig 3: A Different levels of curve subdivision.
After one curve subdivision After two curve subdivision After three curve subdivision

2-2. Implement of mesh subdivision

Mesh subdivision is lead to a smoother and softener mesh. The idea of mesh subdivision is simply shown in Fig. 4.

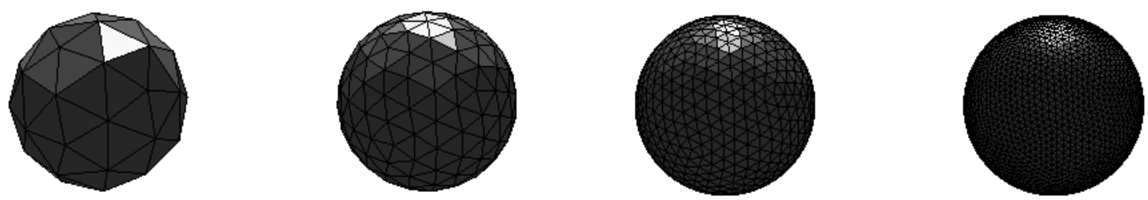

Fig 4: Mesh Subdivision in three steps.

There is different way of mesh subdivision. One which will be explained here is loop subdivision.

Equation 6 shows simply the basic formula of loop subdivision.

$$v'_i = (1 - k\beta)v_i + \sum_{i=0}^{Number\ of\ neighobrs} \beta v_i$$

Equation 6 – Loop subdivision.

In Equation 8 k is the number of neighbors of each vertex and β is calculated in base of k as equation 7.

$$\beta = \begin{cases} \frac{1}{16} & if\ number\ of\ neighbors = 6 \\ \frac{3}{16} & if\ number\ of\ neighbors = 6 \\ \frac{3}{8 * number\ of\ neighbors} & Otherwise \end{cases}$$

Equation 7 – Calculation of β.

So to implement mesh subdivision the following step should be followed:

- For each vertex, find the neighbor vertices.
- By the number of neighbor vertices and equation 7 findβ.
- In a for loop based on the number of neighbor vertices calculate the $\sum$ part of equation 8.
- Add the position of original vertex and return the value of new vertex.

Finally four vertices which be shown in Fig. 5 should be taken. Two of them multiplied by 1/8 and the other by 3/8.

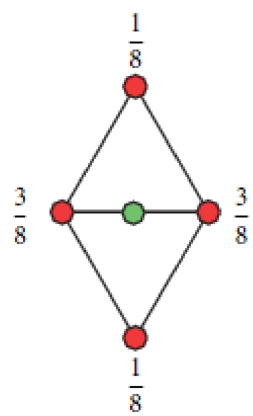

Fig 5: The weights for the new vertex positions.

The output of mesh subdivision shows in fig. 6.

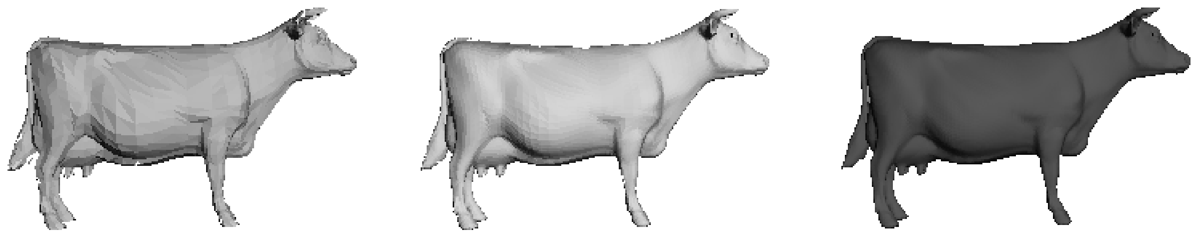

Fig 6: A Different levels of mesh subdivision.

Original mesh | After one mesh subdivision | After two mesh subdivision

2-3. Localize evaluation of the analytical spline

In the current code, a spline is evaluated by the sum of control points multiplied by the basis function. The target of this task is to limit the calculation only to the control points which are supported by the basis function. As it is shown in fig 7 for the b-splines, there are typically 4 basis functions that have non-zero values at any point along the line.

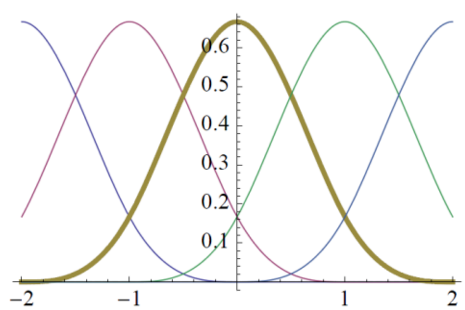

Fig 7: A cubic B-spline basis functions with translated copies of itself.

So for each *t*, calculate the basis function for the pervious and the next three points. All the other calculation is the same as the pervious method.

2-4. Implement a scheme for adaptive mesh

In 2-2, the method for mesh subdivision was described. In this part another method for mesh subdivision will be described.

In simple mesh subdivision all faces -without any priority- divided into smaller parts. But in the adaptive mesh subdivision, based on some rules just certain faces will be divided.

The method will be described here is based on mesh curvature. So a value as a threshold will be chosen (different value for different meshes) and according to this value some faces will be divided and some not.

The output for adaptive subdivision is shown in fig 8.

Fig 8: A Different levels of adaptive mesh subdivision

Original mesh After one subdivision After two subdivision

3. Result

3-1. Curve Subdivision

Curve subdivision will be lead to a smoother line than a straight line. As it clear according to Fig. 3 the first iteration will not acceptable and to get good result two or three iteration should be passed.

3-2. Mesh Subdivision

For each iteration of mesh subdivision the mesh will be smoother and also in each step the time of calculation will be increased. The reason of this is clear. In each iteration the number of triangle which forms the shape will be increased. So in the next step more vertices should be calculated and generated.

3-3. Localize evaluation of the analytical spline

The fact is that the output for this is step has no difference with the original one but the important things which should be mentioned is that the sum of all the output of basic function should be equal to one.

3-4. Implement a scheme for adaptive mesh

By selecting the curvature value as the threshold, the subdivision should be affected at the smooth parts or sharp parts of the mesh.

In this example (fig 7), the sharp parts of mesh was selected to be affected by subdivision. So the horns of cow divided into more triangles and smoother view can be seen in this part and other sharp parts of this mesh.

4. Conclusion

Overall curve subdivision and mesh subdivision will be occurred smoother shape than the original one but should be mention that mesh subdivision is not efficient all the time.

Since I have completed all the *, ** and ***, I should get grade 5.